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20. This surface O is thus a ruled surface of degree eight, every ruling of which meets the Jacobian in three points (or one) and such that three (or one) rulings go through every point on the sextic. The Jacobian is thus a triple line on the surface, three sheets of the surface passing through it. A plane section of the surface will thus give a curve of degree eight with six triple points.

21. This surface O and the Jacobian sextic of the three quadric surfaces will serve to explain the discrepancies in the degrees of reciprocal curves and surfaces. Thus a curve of degree n goes by the transfarmation into a curve of degree $3n$. But since the curve of degree n meets the surface O in $8n$ points, the curve of degree $3n$ meets the sextic in $8n$ points. When therefore, we reciprocate *this* curve of degree $3n$ part of the resulting curve is made up of $8n$ lines of the surface O, and the degree of the resultant curve is thus $9n-8n$ or n, which is the original curve. Similarly for surfaces. Every ruling of O meets a surface of degree n in n points, so that the reciprocal surface of degree $3n$ passes n times through the Jacobian curve. In reciprocating the surface back again we get a surface of degree $9n$ to be sure, but the surface O is n times a part of it. The reciprocal surface is thus of degree $9n-8n$ or n, the original surface.

PROOF OF THE FIRST FORMULA FOR EVALUATING 0/0.

By N. J. LENNES, Columbia University, New York.

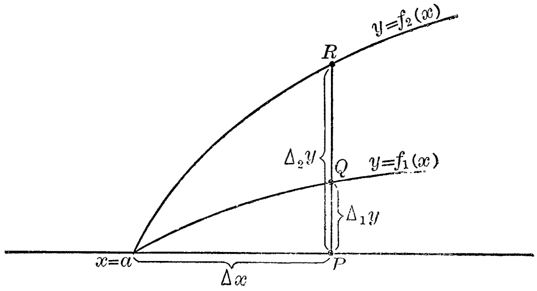
It is well known that many proofs of theorems, while entirely logical and conclusive, afford little or no insight into the theorems proved and certainly furnish no obvious clue to the origin either of the theorem or the proof. On the other hand it is by no means infrequent that one has direct insight into the nature of a theorem or mathematical process and that such insight may be of great value both as an aid to the memory and in judging the true value and uses of the theorem or process while at the same time it may be found impossible, or at least impracticable, to formulate the elements of thought which constitute this insight into a logical proof.

It is believed to be a pedagogical principle of wide application that, other things being equal, the one of two modes of treatment of any topic is to be preferred which affords the more direct and obvious connection between intuitional insight on the one hand and rigorous logical proof on the other. For this reason the proofs given below seem to be worth publishing.

Theorem. *If* $f_1(x)$ *and* $f_2(x)$ *are continuous functions each possessing a derivative at* $x=a$, *and such that* $f_1(a)=0$, $f_2(a)=0$, *while* $f_2'(a)\neq 0$, *then*

$$\operatorname{Lim.}_{x \doteq a} \frac{f_1(x)}{f_2(x)} = \frac{f_1'(a)}{f_2'(a)}.$$

Proof. Using the notation indicated in the figure,



$$f_1'(a)=\underset{\Delta x\doteq 0}{\text{Lim.}}\frac{\Delta_1 y}{\Delta x}$$

$$\text{and } f_2'(a)=\underset{\Delta x\doteq 0}{\text{Lim.}}\frac{\Delta_2 y}{\Delta x},$$

$$\text{and hence } \frac{f_1'(a)}{f_2'(a)}$$

$$=\underset{\Delta x\doteq 0}{\text{Lim.}}\frac{\frac{\Delta_1 y}{\Delta x}}{\frac{\Delta_2 y}{\Delta x}}. \quad \text{But } \frac{\frac{\Delta_1 y}{\Delta x}}{\frac{\Delta_2 y}{\Delta x}}=\frac{\Delta_1 y}{\Delta_2 y}=\frac{f_1(x)}{f_2(x)}.$$

$$\text{Hence, } \frac{f_1'(a)}{f_2'(a)}=\underset{x\doteq a}{\text{Lim.}}\frac{f_1(x)}{f_2(x)}.$$

In case $f_2'(a)=0$ and also $f_1'(a)=0$. the more general theorem is established in the usual manner as follows:*

By the generalized mean value theorem; namely, that if $f_1(x)$ and $f_2(x)$ have derivatives on the interval ab, then there is a value of x, X, on ab, such that

$$\frac{f_1(a)-f_1(b)}{f_2(a)-f_2(b)}=\frac{f_1'(X)}{f_2'(X)}.$$

Since in the case of our theorem we have $f_1(a)=0$ and $f_2(a)=0$ it follows that in the above figure

$$\frac{f_1(b)}{f_2(b)}=\frac{f_1'(X)}{f_2'(X)},$$

or, what is the same thing,

$$\frac{f_1(x)}{f_2(x)}=\frac{f_1'(X)}{f_2'(X)},$$

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* See, for instance, Osgood's *Calculus*, pp. 234, 235.

where X is some point between a and x. Hence, obviously,

$$\mathop{\rm Lim.}_{x\doteq a}\frac{f_1(x)}{f_2(x)}=\mathop{\rm Lim.}_{x\doteq a}\frac{f_1'(X)}{f_2'(X)}.$$

If we now assume $f_2''(a)\neq 0$, we have, by the theorem first proved,

$$\mathop{\rm Lim.}_{x\doteq a}\frac{f_1(x)}{f_2(x)}=\mathop{\rm Lim.}_{x\doteq a}\frac{f_1'(x)}{f_2'(x)}=\frac{f_1''(a)}{f_2''(a)}.$$

Obviously, by the same process, in case $f_2''(a)=0$, but $f_2'''(a)\neq 0$,

$$\mathop{\rm Lim.}_{x\doteq a}\frac{f_1(x)}{f_2(x)}=\frac{f_1'''(a)}{f_2'''(a)},$$

and so on for the higher cases.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

348. Proposed by PROFESSOR L. E. DICKSON, Ph. D., The University of Chicago.

Prove the following relation between Jacobi's symbols:

$$\left(\frac{d}{n}\right)=(-1)^{m(d-1)/2}\left(\frac{d}{2md-n}\right)$$

where d and n are positive odd numbers and $2md>n$.

I. Solution by the PROPOSER.

Replace each symbol by the value given by the generalized reciprocity theorem; then on the right apply

$$\left(\frac{2md-n}{d}\right)=\left(\frac{-n}{d}\right)=(-1)^{(d-1)/2}\left(\frac{n}{d}\right).$$

Hence the relation will be true if